Bayesian Analysis of a NIST Dataset: Interlaboratory Study on Thermal Conductivity

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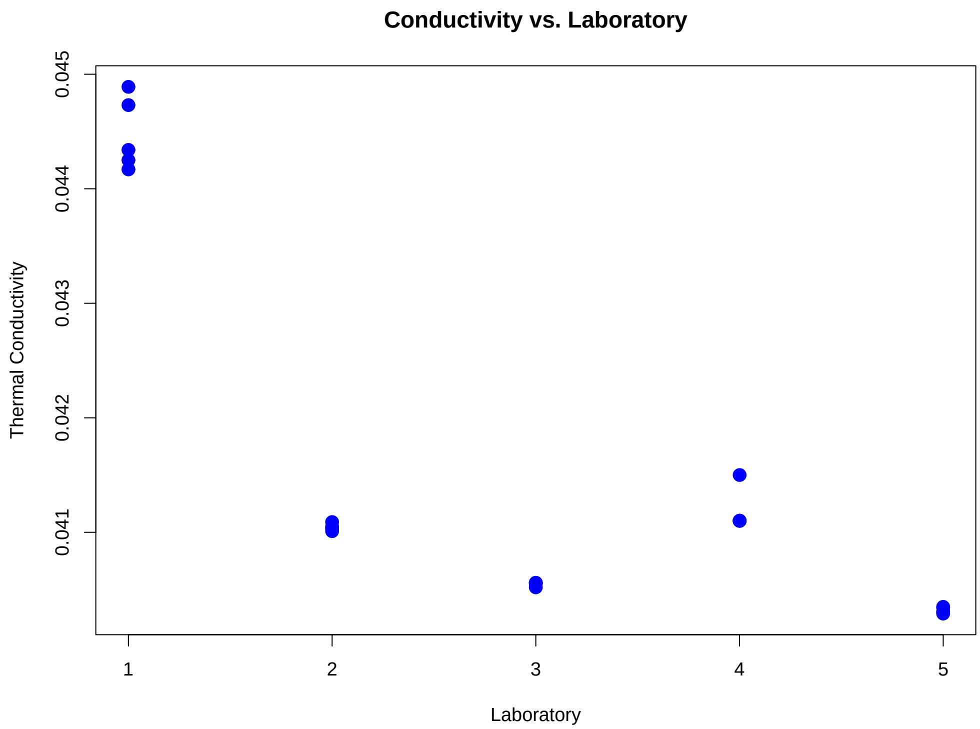
Introduction

- Thermal conductivity measured by 5 labs, with 5 measurements from each lab.

- A Bayesian hierarchical model was fit which allows for both between laboratory variability, and for different within-laboratory measurement uncertainties.

- Results are determined from exact expressions for the posterior distribution by numerical integration.

Data



Note that the variability among the lab means is much greater than the measurement variabilities within labs, and that the different labs have different measurement precisions.

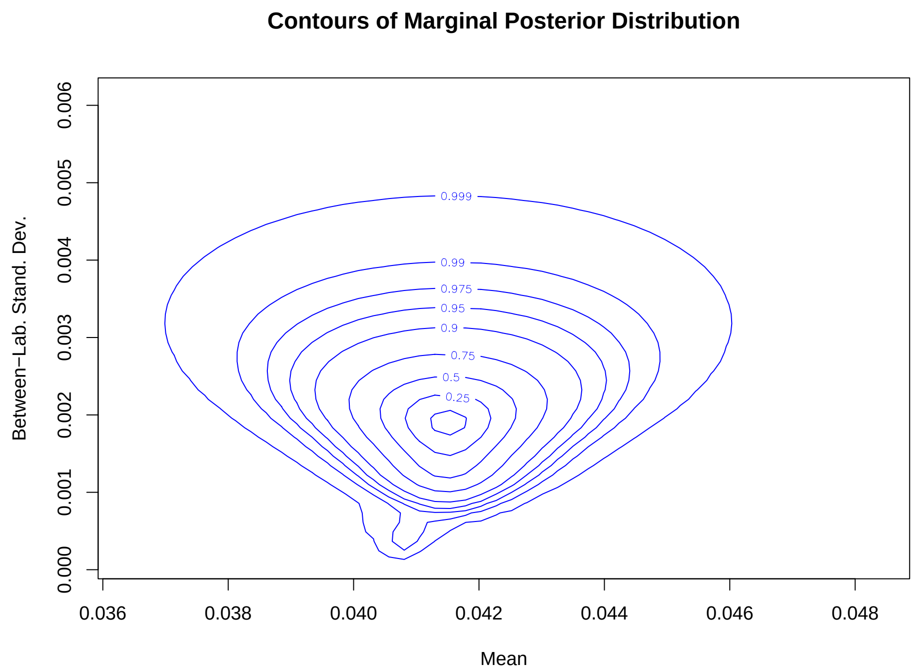
Hierarchical Model With Noninformative Priors

$i = 1, \ldots, k$ indexes laboratories

$j = 1, \ldots, n_i$ indexes measurements

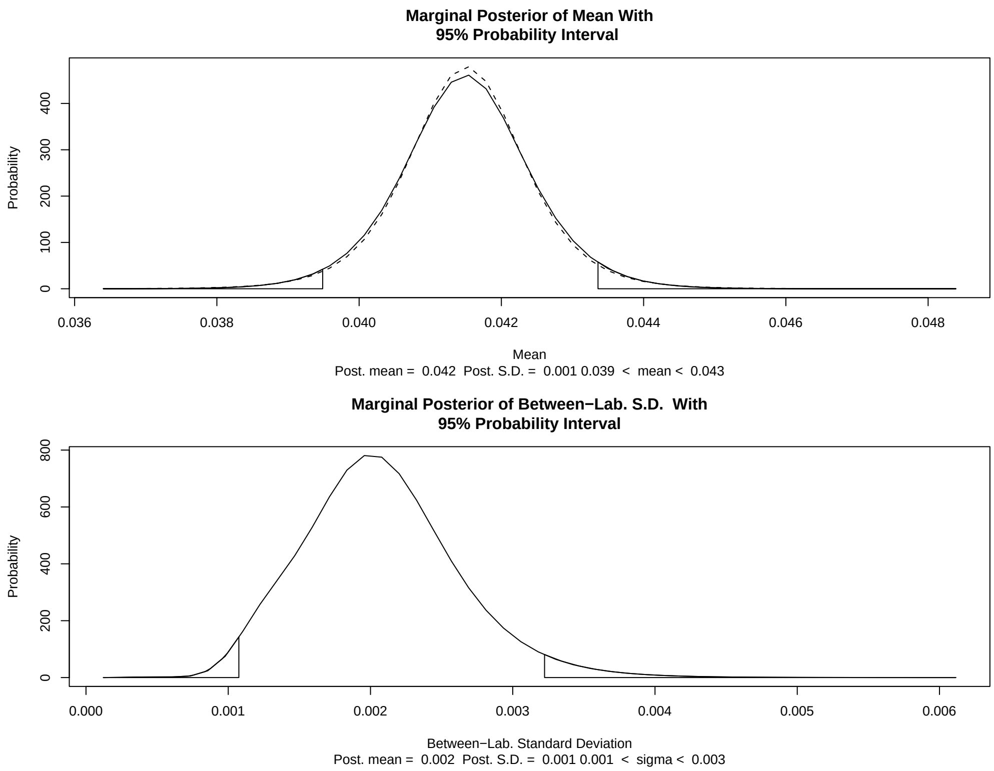
$$
\begin{aligned}
p(x_{ij} | \delta_i, \sigma_i^2) &= \mathsf{N}\,(\delta_\mathsf{i}, \sigma_\mathsf{i}^2) \\
p(\sigma_i) &\propto 1/\sigma_i \\
p(\delta_i | \mu, \sigma^2) &= \mathsf{N}\,(\mu, \sigma^2) \\
p(\mu) &= 1 \\
p(\sigma) &= 1
\end{aligned}
$$

Joint Posterior Distribution of Overall Mean and Between-Laboratory Standard Deviation



Numbers on indicate the probability that the mean and between-lab standard deviation are *both* within the corresponding contour. Note that the between-laboratory standard deviation is definitely nonzero, as one would expect.

Marginal Posterior Distributions for Mean Thermal Conductivity and Between-Laboratory Standard Deviation



These *marginal* distributions were obtained out by integrating out one of the variables in the bivariate distribution displayed previously. The intervals indicated correspond to 95% posterior probability. The broken curve in the top figure indicates t-distribution approximation for posterior mean thermal conductivity.